Physics 131 Some extra problems for those that want them! You're welcome! Dr. AP

Kinematics (Gr. *kinesis* motion) is the description of motion; dynamics (Gr. *dynamikos* powerful) is an attempt to understand the *cause* of motion – i.e., forces!

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1. Gravity is a "downward" force in our daily lives. For most biologists (with the exception of xenobiologists!), it is not worth noting that the full form of Newton's gravity is $\propto mM/R^2$; that is, the force of attraction is proportional to the two masses considered and inversely proportional to the square of the distance between them (it's crucial to understanding gravity between planets and moons and such on the large scale). Life on Earth is pretty much confined to the surface and the atmosphere, a thin layer like a shell that's pretty much the same distance from the center of the Earth whether at the bottom of the Mariana Trench 6 km down from mean sea level in the Pacific Ocean or the thinner regions of the stratosphere 20 km up from mean sea level. The radius of the Earth is about 6400 km for comparison! So we can get away with treating the M/R^2 as constant (specifically, since $F = GmM/R^2$, we can lump the stuff that's essentially constant into one constant "g": $F = mg$; $g = GM/R^2$. For all objects in your daily life from planes in the sky to submarines in the ocean, from a grapefruit to an apple, gravity on that object is just proportional to the mass of that object with proportionality constant "g".
    a. What is the ratio of the force of gravity from the Earth on an apple vs. a grapefruit?
    b. If we were holding an apple in the left hand and a grapefruit in the right and let go of them at the same time, why do they accelerate the same if the force of gravity is different for both?
    c. If I drop a feather, it tends to accelerate briefly but then fall at a constant speed. As it's falling, why will the magnitude of the force of air friction on the feather never be more than the magnitude of the force of gravity? Why does the feather keep falling if the two forces are equal in magnitude?

Don't make this problem harder than it is. Since it's not specified, you can assume these questions are from daily life here at the surface of the Earth under normal atmospheric conditions.

a. Treating them as spheres (spherical cow approximation!), an apple is about ½ the diameter (i.e., ½ the radius) of a grapefruit, so it's around 1/8th the volume $V_g/V_a = (4\pi/3)R_g^3/[(4\pi/3)R_a^3] = R_g^3/R_a^3$. They're both made out of mostly water, so presumably the densities are comparable $\rho_g = m_g/V_g \approx \rho_a = m_a/V_a \approx \rho_{water}$. This means the grapefruit is about 8 times the mass of an apple $m_g/m_a = V_g/V_a = 8$ (this is a shaky comparison if the densities *aren't* comparable but as with all such problems, just tell us what your assumptions are!). So since "…gravity on that object is just proportional to the mass of that object," we get $F_{Earth\ on\ g}/F_{Earth\ on\ a} = m_g/m_a = 8$, that is the force of gravity is eight times larger on the grapefruit. Long way to get there, but explanations like this convince the grader that you understand the problem!

b. Newton's 2nd law says that the net force (the vector sum of all forces) on an object causes it to accelerate in proportion to its mass, $\vec{F}_{net} = m\vec{a}$, so if the *only* force on the object happens to be gravity $\vec{F}_{net} = \vec{F}_{grav}$, the mass is on both sides and cancels out! $mg = ma$

c. Notice that "if" from our answer to part b.: "if the only force on the object…" In the case of a feather, the air resistance is non-negligible. The reason it falls at a constant speed is that the air resistance is proportional to the speed $\vec{F}_{air\ resistance} \propto -\vec{v}$ (the minus sign here is directional!). So as it "briefly accelerates" the magnitude of the force of air friction increases with increasing speed. At some point the force of air friction equals the force of gravity (presumably the only forces on the feather) so the net force drops to zero. At that point, it keeps falling since a net force of zero means no *change* in velocity, i.e., no acceleration. We're at Newton's 1st law: an object in motion continues in motion if no net force is acting on it. The first law is secretly a special case of the second law $F_{net} = ma$ where since the net force is zero, so is a.

2. Aristotelian physics says that an object's natural state of motion is at rest. Why did it take so long for people (Galileo, Newton and so forth) to figure out this is wrong? Start with speculation on why Aristotle's ideas are backed up by evidence.

Our daily experience shows that things always seem to come to rest eventually (except the heavens rotating around the Earth!), and that some kind of motivated force is needed to make things do otherwise. So the evidence is, at brief glance, supportive of Aristotle's ideas. The problem is that people did not have a useful, predictive mechanism for taking friction into account. Once you posit that friction *between* an object and its environment is the motive force for making things come to rest, it becomes easier to see why the moons and planets dance eternally in the heavens but can still be subject to the laws of Newton. And we can start to better see how to account for the motion (specifically the *change* in motion = "acceleration") of an object being due to the superposition (sum) of all the forces on that object.

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3. A pedestrian example, all puns intended…
   You walk to class every day (well, some of you), so how do you get there?
   a. Approximate your pace as a "constant" speed of 2 m/s. In that case, what is the average horizontal force on you and from what source/sources?
   b. You don't actually move at a constant speed of course. Describe your motion in more detail and draw a free body diagram (FBD) for the "start" of one step and another free body diagram for the "end" of that step; identify all the forces on you in each case (treat your body as a simple "point mass"). Be sure to indicate on your FBD which way is forward.
   c. On an icy day, you "skate" on both your shoes (i.e., neither one leaves the ground) on the way to class. Draw the same two diagrams that you drew in part c. but now for the "beginning" and the "end" of a single skating step. Even though you should still treat your entire body as a point mass, identify the forces on you from the ground by which shoe is receiving that force: the "back" foot or the "front" foot. Comment on why ice skaters align their skates one way to push to speed up, but another way to glide.
   d. Comment on "static" friction: does it always oppose motion?

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a. This is a trick question! Constant speed means zero (average) horizontal forces. Your "push" from part a. pushes you forward, but your other foot lands pushing back until your body gets on the "other side" of it and can push forward on the next step. If you're walking at a constant speed *on average,* then the forces forward and back much balance *on average.*
b. For both FBDs, the vertical forces look the same: gravity from the Earth on you pointing down, contact force from the ground on you pointing up (each of equal magnitude). Since you start at rest and then accelerate forward, something must be pushing you: friction from the Earth ***on you pointing in the direction of travel (forward)*** – this is the key point that so many students get confused about. As you walk to class, you know you push on the ground with your foot and can picture the foot forcing the ground *backwards* as if the Earth were a gigantic treadmill. The problem is, we're asking about *your* motion (not the Earth's negligible response to your push) and are only concerned with the forces ON YOU. Newton's 3rd law says that your push on the Earth is the same magnitude (but opposite direction) as the force of the Earth on you. So you push on the ground, the ground pushes back (in the forward direction) and you accelerate forward. For the 2nd FBD, the foot lands and can decelerate you somewhat, so friction points *backward.* In reality your body never quite reaches a speed of zero, but swings over each foot – this is a complication that's impossible to see if you use the "point mass" simplification.
c. The main difference in the diagrams is that the back foot is pushing forward with a greater force than the front foot is pushing back at first; then the other way round as the step "ends." Skaters push the ice with the long blade oriented as much as 90 degrees from the direction of travel: this gives them a larger static friction surface to push against. The other foot is oriented parallel to the direction of travel to minimize friction.
d. No. Again, this is a tricky thing for students to wrestle with. Students want simple absolute correlation like "static friction always points in the direction of travel" or "kinetic ("sliding") friction always opposes the motion. But it's not too hard to give counterexamples to either of these: like the walking example above!

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4. You push a cart of mass 45 kg (100 lb) with wheels across a horizontal floor with no sliding friction (and ignore the sliding friction in the wheels' bearings/axle, etc.) starting from rest and get it up to a speed of 2 m/s in 5 seconds.
   a. What's the average acceleration of the cart?
   b. Assuming that the average acceleration is also the *actual* acceleration in that time period, what is the position of the cart after those 5 seconds?
   c. How hard was your average push against the cart in that time period?
   d. If you stopped pushing right at that 5 second mark, what does the cart do?
   e. Starting at that 5 seconds, how hard would you have to pull (on average) to get the cart to stop in 1 second?
   f. Does your answer to part e. change if you waited to start pulling until the 6th second?
   g. Based on the setup given and e., graph the cart's acceleration, velocity and position, the force of you pushing on the cart and the force of the cart pushing on you in a "stack" of five graphs such that the tick marks on the time axes match up vertically. Time should run from $t = 0$ when you started pushing the cart from rest until $t = 6$ when the cart came to rest.

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a. Remember how the average velocity was total displacement (final position – initial position) divided by the time period in question (final time – initial time)? Same idea here:
$\langle a\rangle = (v_{final} - v_{initial})/(t_{final} - t_{initial}) = (2\text{ m/s} - 0\text{ m/s})/5\text{ s} = 0.4\text{ m/s}^2$

b. This means that $a(t) = \langle a\rangle = 0.4\text{ m/s}^2$ so

$$v(t) = v_0 + \int a(t)dt = 0 + 0.4\text{ m/s}^2 t = (0.4\text{ m/s}^2)t$$

and

$$x(t) = x_0 + \int v(t)dt = 0 + \frac{1}{2}0.4\text{ m/s}^2 t^2 = (0.2\text{ m/s}^2)t^2$$

so

$$x(5\text{ s}) = (0.2\text{ m/s}^2)(5\text{ s})^2 = 5\text{ m}$$

There are many ways to do this given that we know it's a constant acceleration!

c. Since the cart is presumed to undergo uniform acceleration, and your push is the only relevant horizontal force, then $\vec{F}_{net\ on\ cart} = \vec{F}_{your\ push\ on\ cart} = m\vec{a}_{cart} = (45\text{ kg})(0.4\text{ m/s}^2)\hat{\imath} = 18\text{ N}\hat{\imath}$ where $\hat{\imath}$ is presumed to mean "forward" in the direction the cart is going – this is seemingly picky, but useful for e. below.

d. No other forces are mentioned and friction is specifically negligible in the problem. So it keeps going at 2 m/s until it experiences another force (hits a wall, grabbed by someone else, goes down a ramp instead of beingon a flat floor, etc.).

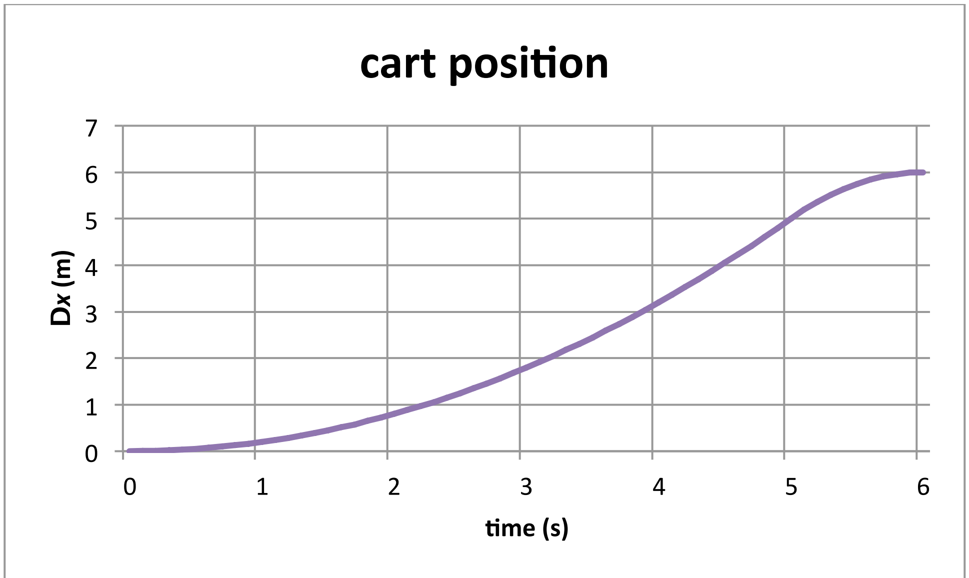
e. This is just the opposite of the push setup, but faster. So we drop from 2 m/s to 0 in 1 second, for an average acceleration of $\langle a\rangle = (v_{final} - v_{initial})/(t_{final} - t_{initial}) = (0\text{ m/s} - 2\text{ m/s})/1\text{ s} = 2\text{ m/s}^2$ and we're *pulling* against the motion so I'll use vector notation explicitly for completeness:

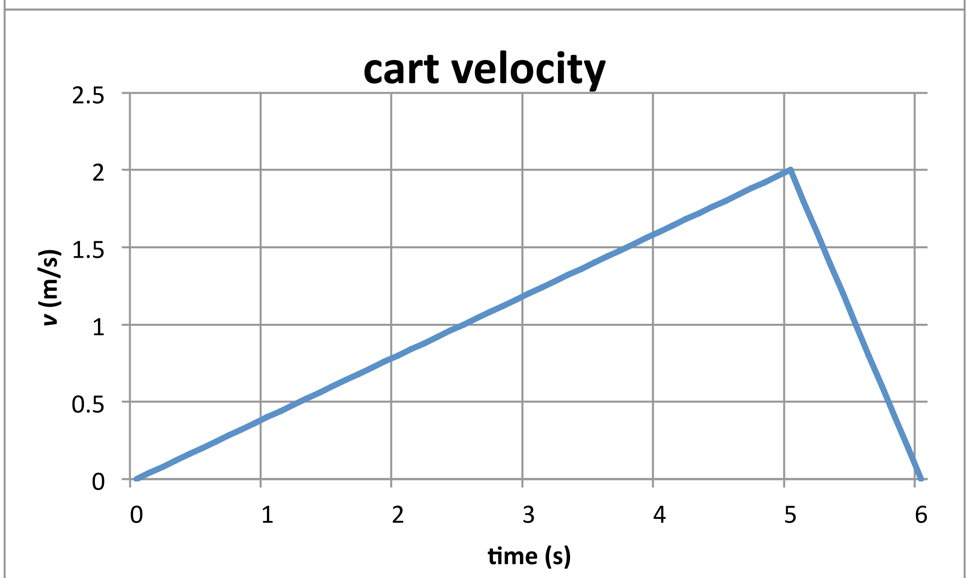
$$\vec{F}_{net\ on\ cart} = \vec{F}_{your\ pull\ on\ cart} = m\vec{a}_{cart} = -(45\text{ kg})(2\text{ m/s}^2)\hat{\imath} = -90\text{ N}\hat{\imath}$$

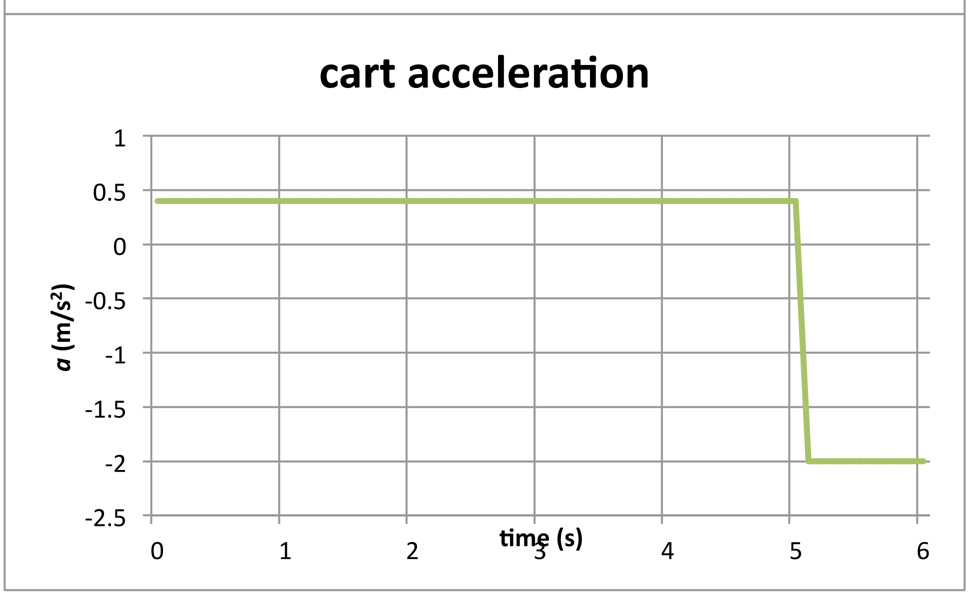
where the minus sign indicates that the acceleration happens to point opposite to the original choice of positive direction. At this point, it's useful to ask you if you were picturing actually doing this with a shopping cart at a grocery store, because the numbers are roughly relevant, and the experience (pushing and pulling) might be worth trying out!

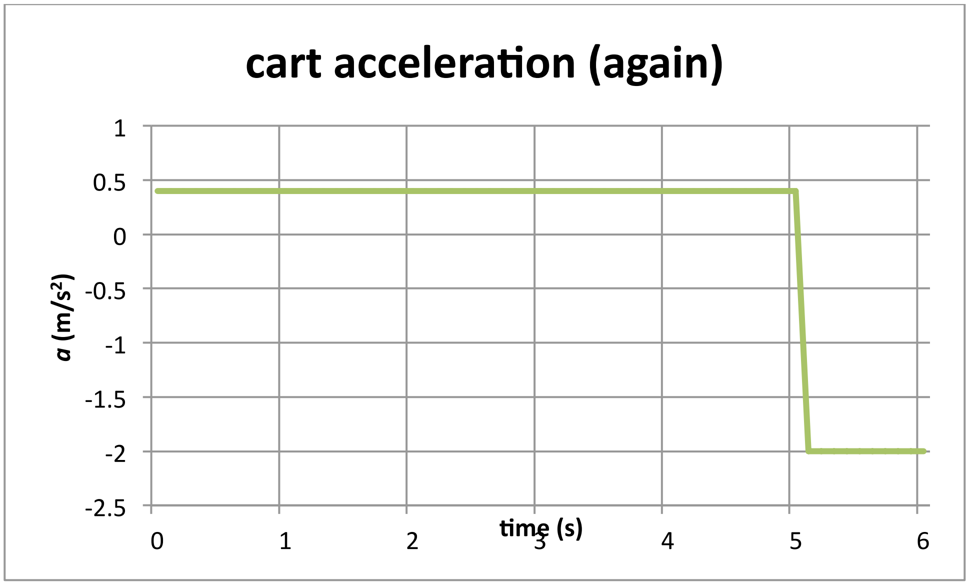
f. No. Presumably the cart trundles along at a constant speed – you have to pull with 90 N opposite to the speed of 2 m/s to get it to stop in 1 second no matter which second we're talking about.

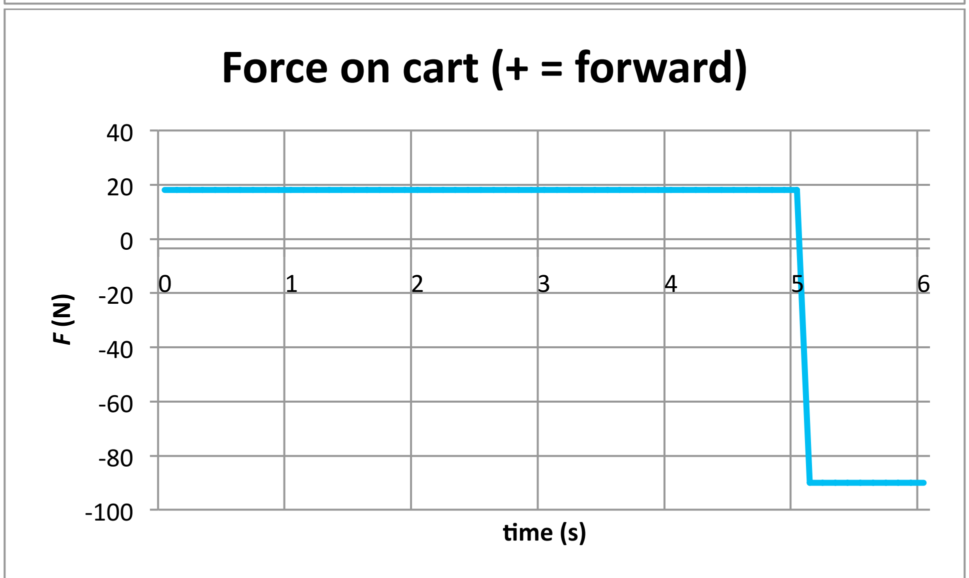
g. See below – stacked! Note especially how there's no qualitative difference between the cart's acceleration graph and net force graph (which just happens to be just the push/pull from you!), just a quantitative one since $\vec{F}_{net\ on\ cart} = \vec{F}_{your\ push\ or\ pull\ on\ cart} = m\vec{a}_{cart}$. Note also 3rd law forces the last two graphs to look identical *EXCEPT* for sign!

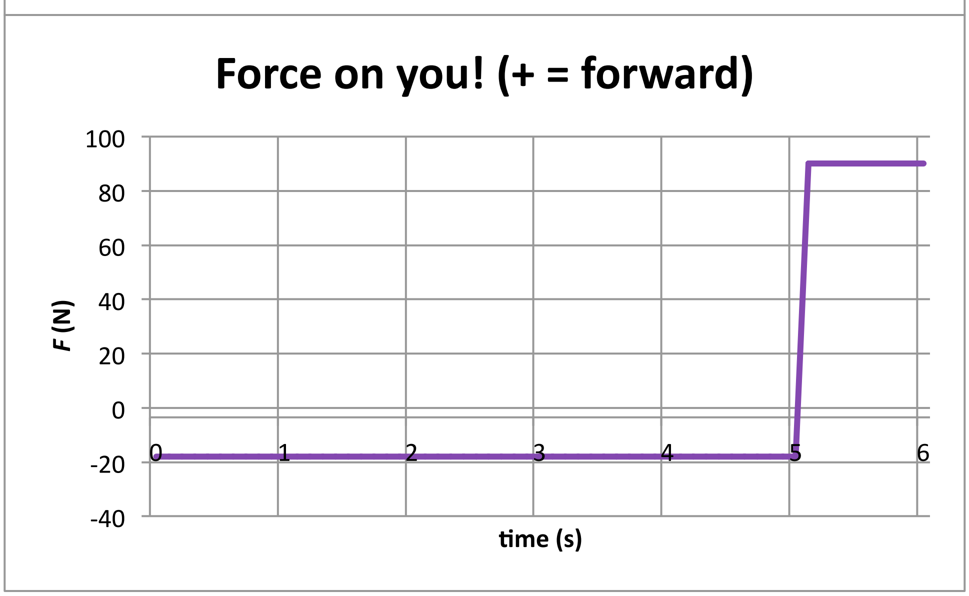

cart position
7
6
5
4
3
2
1
0
Dx (m)
0
1
2
3
4
5
6
time (s)


cart velocity
2.5
2
1.5
1
0.5
0
v (m/s)
0
1
2
3
4
5
6
time (s)


cart acceleration
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
a (m/s²)
0
1
2
3
4
5
6
time (s)


cart acceleration (again)
a (m/s²)
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
0
1
2
3
4
5
6
time (s)


Force on cart (+ = forward)
F (N)
40
20
0
-20
-40
-60
-80
-100
0
1
2
3
4
5
6
time (s)


Force on you! (+ = forward)
F (N)
100
80
60
40
20
0
-20
-40
0
1
2
3
4
5
6
time (s)